Engineering Notes

ENGINEERING NOTES are short manuscripts describing new developments or important results of a preliminary nature. These Notes cannot exceed 6 manuscript pages and 3 figures; a page of text may be substituted for a figure and vice versa. After informal review by the editors, they may be published within a few months of the date of receipt. Style requirements are the same as for regular contributions (see inside back cover).

Direct Velocity Feedback Control of Large Space Structures

Mark J. Balas*
Bolt, Beranek, and Newman, Inc., Cambridge, Mass.

Introduction

THE construction and operation of large space structures (LSS) poses some new and challenging active control problems. These structures are extremely mechanically flexible because of their large size, low rigidity, and the low natural damping available in lightweight construction materials. Therefore, active vibration suppression becomes essential to the construction and operation of these LSS.

In this Note, a method is presented for active vibration suppression called direct velocity feedback (DVFB) because the output signals from velocity sensors are electronically multiplied by gains and these signals are directly fed back to collocated force actuators. In contrast to previous LSS control systems,[1,2] where a low number of critical vibration modes is controlled and spillover into the residual modes can destabilize some of these modes (unless appropirate compensation is used), the DVFB method will guarantee that all vibration modes remain stable when the active control is in operation. Also, DVFB is extremely simple compared to other approaches. However, lest it be regarded as a panacea for all LSS control problems, DVFB has several limitations and it is the intent of this Note to point out and explain these limitations so that DVFB may be used appropriately in the LSS control hierarchy.

Consider LSS described by the generalized wave equation:

$$m(x)u_{tt}(x,t)+2\xi A^{1/2}u_t(x,t)+Au(x,t)=F(x,t) \tag{1}$$

which relates the displacements $u(x,t)$ of the equilibrium position of a flexible structure Ω (a bounded open connected set with smooth boundary $\partial\Omega$ in n-dimensional space R^n) to the applied force distribution $F(x,t)$. The mass density $m(x)$ is a positive function of the location x on the structure. The change of variables $u(x,t)\rightarrow u(x,t)/m(x)^{1/2}$ eliminates $m(x)$ without changing the properties of Eq. (1) and, henceforth, assume $m(x)\equiv 1$. The non-negative real number ξ is the damping coefficient of the structure; it is quite small for LSS. The operator A is a symmetric, non-negative, time-invariant differential operator with compact resolvent and a square root $A^{1/2}$. The domain $D(A)$ of A contains all sufficiently differentiable functions which satisfy the appropriate boundary conditions for the LSS. $D(A)$ is dense in the Hilbert space $H\equiv L^2(\Omega)$ with the usual inner product $(.,.)$ and associated norm $\|\cdot\|$.

The control forces are provided by M point-force actuators,

$$F(x,t)=\sum_{i=1}^{M} b_i(x)f_i(t) \tag{2}$$

Received May 16, 1978; revision received Oct. 10, 1978.

Index categories: Spacecraft Dynamics and Control; Structural Dynamics.

*Senior Scientist, Control Systems Dept. Member AIAA.

where the actuator influence functions $b_i(x)$ are in H and approximate the Dirac delta functions $\delta(x-x_i)$. The displacements and velocities are measured by P point-sensors:

$$y(t)=\bar{C}u(x,t)+\bar{D}u_t(x,t) \tag{3}$$

where $y_j(t)=c_ju(z_j,t)+d_ju_t(z_j,t)$ with c_j, d_j fixed non-negative real numbers, z_j the sensor locations, and $j=1,2,\ldots,P$. The energy in the undamped system ($\xi=0$) is

$$E(t)\equiv E(u,u_t)=\tfrac{1}{2}(\|u_t\|^2+\|A^{1/2}u\|^2) \tag{4}$$

which generates the energy (semi) norm given by $\|(u,u_t)\|_E\equiv E(u,u_t)^{1/2}$. The systems described by Eqs. (1-3) are representative of the generic problems of LSS.

It is well known (Ref. 3, p. 277) that the spectrum of A contains only isolated eigenvalues λ_k with corresponding orthogonal eigenfunctions $\phi_k(x)$ such that

$$0\leq\lambda_1\leq\lambda_2\leq\lambda_3\leq\ldots$$

and

$$A\phi_k=\lambda_k\phi_k \qquad A^{1/2}\phi_k=\lambda_k{}^{1/2}\phi_k$$

The eigenfunctions are the mode shapes of the structure and the mode frequencies are $w_k=\lambda_k{}^{1/2}$. The solution of Eq. (1) can be expanded in the mode shapes as

$$u(x,t)=\sum_{k=1}^{L}u_k(t)\phi_k(x) \tag{5}$$

where, in theory, L is infinite but, in engineering practice, L can be large and finite since the highest frequency modes will be difficult to excite with the active controller. The mode amplitudes $u(t)=[u_1(t),\ldots,u_L(t)]^T$ satisfy, from Eqs. (1) and (2),

$$\ddot{u}(t)+2\xi\Lambda^{1/2}\dot{u}(t)+\Lambda u(t)=Bf(t) \tag{6}$$

where $\Lambda^{1/2}\equiv$ diagonal $[w_1,\ldots,w_L]$, $\Lambda\equiv(\Lambda^{1/2})^2$, $f(t)=[f_1(t),\ldots,f_M(t)]^T$, and B is $L\times M$ matrix with entries $b_{kl}=\phi_k(x_l)$. The sensor Eq. (3) becomes

$$y(t)=D_1Cu(t)+D_2C\dot{u}(t) \tag{7}$$

where $D_2\equiv$diagonal $[c_1,\ldots,c_p]$, $D_2\equiv$diagonal $[d_1,\ldots,d_p]$ and C^T is $N\times P$ matrix with entries $\phi_k(z_l)$.

Main Results

The following assumptions are made: 1) the number of sensors is equal to the number of actuators (i.e., $P=M$); 2) only velocity sensors are used [i.e., in Eq. (7), $D_1=0$ and $D_2=I_p$]; and 3) the actuators and sensors are collocated [i.e., in Eq. (7), $C=B^T$]. With these assumptions, the sensor Eq. (7) becomes

$$y(t)=B^T\dot{u}(t) \tag{8}$$

The method of direct velocity feedback (DVFB) is obtained with a control law of the following form:

$$f(t)=-Qy(t) \tag{9}$$

where Q is a $P\times P$ symmetric, non-negative definite gain matrix. The choice of a specific gain matrix Q is left open for the moment. The open-loop system is given by Eq. (6) with no control applied (i.e., $f=0$) and the closed-loop system is the same equation with the DVFB control f given by Eq. (9) along with the sensor Eq. (8). The following stability result is obtained.

Theorem 1: The closed-loop system with DVFB is energy dissipative, i.e., $E(t)$ is decreasing, and if there are no zero frequencies in the open-loop system (i.e., Λ is positive definite) then the closed-loop system is stable for any $\xi \geq 0$ and asymptotically stable, if either 1) $\xi > 0$ or 2) $\xi \geq 0$ and $BQ^{1/2}$ is nonsingular.

Proof: From Eqs. (4) and (5),

$$E(t) = \tfrac{1}{2}(\dot{u}^T\dot{u} + u^T\Lambda u)$$

and, using Eq. (6),

$$\dot{E}(t) = \dot{u}^T\ddot{u} + \dot{u}^T\Lambda u = -2\xi\dot{u}^T\Lambda^{1/2}\dot{u} + \dot{u}^TBf$$

But, from Eqs. (8) and (9),

$$f = -QB^T\dot{u}$$

Therefore,

$$\dot{E}(t) = -\dot{u}^T[2\xi\Lambda^{1/2} + BQB^T]\dot{u}$$

However, $\Lambda^{1/2} \geq 0$ and $BQB^T \geq 0$, and so $\dot{E}(t) \leq 0$ or $E(t)$ is dissipative. If $\Lambda > 0$, then $E(t)$ is positive and $\dot{E}(t)$ is negative on all nonzero states $(u, \dot{u})$ and, consequently, $E(t)$ is a Lyapunov function.[4] Thus, the closed-loop system is stable, for any $\xi \geq 0$, and it is asymptotically stable if the matrix $W = 2\xi\Lambda^{1/2} + BQB^T$ is positive definite (i.e., $E(t)$ is decreasing). The matrix W is positive definite if either condition 1 or 2 is true and, therefore, the stability result is obtained.

The basic idea of Theorem 1 is well known in the mathematical literature on distributed parameter systems,[5] but does not seem to have penetrated the engineering literature on LSS. It should be noted that condition 2 of Theorem 1 is very difficult to achieve in practice because it requires that the number of sensors P equals the total number of modes L in the LSS. However, condition 1 is reasonable in that some damping is present in all LSS; the requirement that no rigid-body (zero frequency) modes be present can be alleviated partially by the following result.

Theorem 2: If Λ contains zero frequencies, where

$$\Lambda = \begin{bmatrix} 0 & 0 \\ 0 & \Lambda_2 \end{bmatrix} \qquad \Lambda_2 > 0$$

then a sufficient condition for stability of the nonzero frequency modes is that the energy in the zero frequency modes remains constant under feedback control (in particular, that the actuators do not excite the zero frequency modes). If the nonzero frequency modes are damped, then they are asymptotically stable.

Proof: Separate $E(t) = E_1(t) + E_2(t)$, where $E_1(t)$ is the energy in the zero frequency modes and $E_2(t)$ is the energy in the nonzero frequency modes. From Theorem 1, $\dot{E}(t) \leq 0$ or $\dot{E}_2(t) \leq -\dot{E}_1(t)$. Now, if $E_1(t)$ is constant, then $\dot{E}_2(t) \leq 0$ and, thus, $E_2(t)$ is a Lyapunov function for the nonzero frequency mode subsystem which makes it asymptotically stable. It is easy to see that $E_1(t)$ is constant for the special case where the actuators do not excite the zero frequency modes.

The important limitation of this result is that the energy in the rigid-body modes must be conserved while the controller is in operation; this can be achieved in some practical situations with actuators that do not excite rigid-body modes, e.g. member dampers, or with pairs of actuators where the rigid-body excitation is balanced out to zero. Nevertheless, DVFB must be used with care in LSS with any zero-frequency modes. It may be possible to introduce some position feedback control to shift the zero frequency modes away from zero; however, such feedback control alters the stiffness matrix Λ into a new $\tilde{\Lambda}$ with no zero frequencies and this new $\tilde{\Lambda}$ must be symmetric for the application of Theorem 1.

Finally, the choice of feedback gains Q needs to be addressed. Since Theorems 1 and 2 indicate stability regardless of the choice of Q (as long as Q is non-negative definite), the feedback gains become a design parameter. They can be chosen to improve the damping in any subset of critical vibration modes with the knowledge that the controller will not destabilize any part of the system (as long as the appropriate conditions of Theorems 1 or 2 are satisfied). However, the actual pole locations of the closed-loop system will have to be determined from a rather large symmetric eigenvalue calculation:

$$\det(s^2 + Ws + \Lambda) = 0 \tag{10}$$

where $W = 2\xi\Lambda^{1/2} + BQB^T$ and there are L conjugate pairs of poles.

Conclusions

DVFB does have limited application to LSS control; it can be used to supress vibrations in LSS under the following restrictions: 1) an equal number of collocated force actuators and velocity sensors are used; 2) the feedback gain matrix Q is non-negative definite; and 3) if zero frequency modes exist, the actuators must maintain constant energy in these modes (e.g., the actuators must not excite these modes). Under these restrictions, the DVFB controller cannot destabilize any part of the system no matter what the choice of feedback gains, and the stability margin of the closed-loop system will never fall below that of the open-loop system. However, the exact pole locations produced by DVFB must still be found from a large symmetric eigenvalue problem. This Note does not suggest that DVFB is suitable for the many other more complex LSS control problems, such as accurate pointing or attitude control, but it may prove useful as an adjunct to other controllers in dealing with the complex hierarchy and interdependence of LSS control problems.

References

[1] Balas, M., "Active Control of Flexible Structures," *Journal of Optimization Theory and Applications*, Vol. 25, July 1978, pp. 415-436.

[2] Skelton, R. and Likins, P., "Orthogonal Filters for Model Error Compensation in the Control of Nonrigid Spacecraft," *Journal of Guidance and Control*, Vol. 1, Jan.-Feb. 1978, pp. 41-49.

[3] Kato, T., *Perturbation Theory for Linear Operators*, Springer-Verlag, New York, 1966.

[4] LaSalle, J. and Lefschetz, S., *Stability by Lyapunov's Direct Method with Applications*, Academic Press, New York, 1961.

[5] Russell, D., "Linear Stabilization of the Linear Oscillator in Hilbert Space," *Journal of Mathematical Analysis and Applications*, Vol. 25, 1969, pp. 663-675.

Vector Methods in Homing Guidance

Malcolm J. Abzug*
Aeronautical Consultant, Pacific Palisades, Calif.

Nomenclature

g	= acceleration of gravity
i,j,k	= unit vectors along inertial reference axes
n	= PN guidance load factor
n_l	= PN load factor component due to line-of-sight rate
R	= interceptor to target current range
R_p	= predicted range vector

Received Jan. 5, 1979.

Index categories: LV/M Guidance; Guidance and Control.

*Fellow AIAA.